IMAGE RECONSTRUCTION FOR THE MEGA TELESCOPE

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ABSTRACT

The Compton scattering and pair creation telescope MEGA detects gamma-rays in the energy range from 400 keV up to at least 50 MeV. Its multi-dimensional response presents a challenge for image reconstruction: The large amount of measured parameters and the geometry result in a response, which depends on incidence angle, energy, Compton scatter angle, direction and energy of scattered electrons, etc. Moreover the image reconstruction algorithm has to incorporate different event types into one image (tracked und untracked Compton events as well as pair events) and has to cope with high background conditions. An algorithm, which meets this challenges, is the iterative List-Mode Maximum-Likelihood Expectation-Maximization algorithm. The adoptions of this algorithm to the needs of the MEGA telescope are shown along with selected results.

Key words: gamma-ray astronomy; Compton telescope; image reconstruction; list-mode.

1. INTRODUCTION

The tracking Medium Energy Gamma-ray Astronomy telescope MEGA has been developed at the Max-Planck-Institute for extraterrestrial Physics (Garching, Germany). It is designed to detect gamma-rays in the energy range from 400 keV up to at least 50 MeV via Compton scattering and pair creation (Kanbach et al., 2003).

MEGA consists of a tracker, in which the primary Compton scatter or pair creation event takes place, and a calorimeter, which absorbs and measures the secondary particles. The basic design and the measurement principle of the telescope are shown in figure 1. A prototype for MEGA has been built and calibrated, which has 12-times less volume than the satellite version under study (Andritschke et al., 2004).

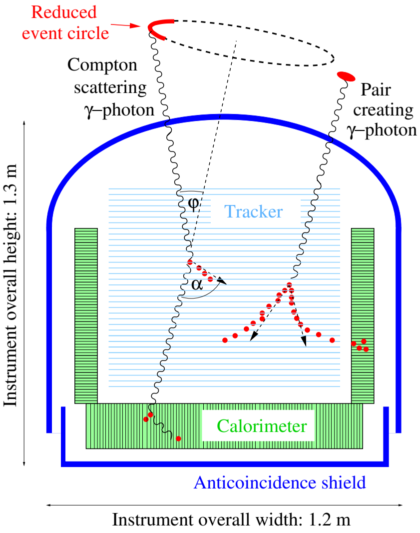

Figure 1. Baseline design and measurement principle of the full MEGA telescope. The recoil electrons from Compton scattering as well as the pair creation products are tracked in a stack of Silicon strip detectors and the secondary particles are stopped in the surrounding CsI calorimeter.

2. DATA-SPACE AND SELECTION OF THE IMAGING ALGORITHM

The measurement process of any telescope can be described in the following way:

$$D(\vec{d}) = T(\vec{d}; \chi, \psi) \times I(\chi, \psi) + B(\vec{d}) \tag{1}$$

Photons emitted from the image-space I at position (χ, ψ) undergo the measurement process T and, after adding some background B, are measured with the parameters $\vec{d}$ in the data-space D. The task of image reconstruction is to determine the image $I(\chi, \psi)$ while the measurement $D(\vec{d})$ is given. Since no unique solution for this inversion problem exists, iterative approaches for the reconstruction of the image have to be chosen.

For a modern tracking Compton telescope the data-space D consists at least of the following parameters:

the direction of the scattered gamma-ray (χ_g, ψ_g) and the Compton scatter angle φ as well as the direction of the recoil electron (χ_e, ψ_e) and its scatter angle ϵ. The two scatter angles have been chosen instead of the energy of the scattered gamma-ray E_g and recoil electron E_e, since these parameters are a better description of the measurement process. The minimum number of bins for the data-space and response matrix for the MEGA telescope, which would not lead to the loss of a significant amount of information, is given in Table 1.

Measured parameter	Size [bins]
Dir. scat. γ (χ_g, ψ_g)	$2 \cdot 360 \times 2 \cdot 180$
γ scatter angle φ	4×120
Dir. recoil e^- (χ_e, ψ_e)	180×45
e^- scatter angle ϵ	90
Sum data-space	$\sim 10^{14}$
Field of view (≤ 2 sr)	$2 \cdot 160 \times 2 \cdot 160$
Sum response matrix	$\sim 10^{19}$

Table 1. Minimum size of the MEGA data-space and response matrix for a sky image (far field). The number of bins for the measured parameters is the minimum *number which does not lead to the loss of a* very significant *amount of information.*

The final size of the data-space ($\sim 10^{14}$ bins) and the response matrix T ($\sim 10^{19}$ bins) are far beyond the possibilities of any state-of-the-art personal computer – it can neither be filled via simulations, nor can it be kept in RAM for image reconstruction (the data-space would need $\sim$ 100 TB). Moreover, the proposed data-space is already a strong simplification: it does not take into account the different calorimeters with their different energy resolutions, it ignores multiple interactions and the distance between the two interactions, which influences the angular resolution through the spatial detector resolution. Additionally, for near field measurements with the MEGA prototype another parameter is necessary, the position (x, y, z) of the first interaction, which easily increases the data-space by another factor of 10^6.

Therefore all types of imaging algorithms which use a binned data-space, like they have been used for COMPTEL or INTEGRAL, are completely ruled out. As a consequence a list-mode based algorithm has to be used. In this type of algorithms no binned data-space is used, but a mere list of events (Barret et al., 1997). Thus the size of the data-space is no longer fixed, but proportional to the number of measured events. As long as the number of events is significantly smaller than the number of data-space bins, list-mode algorithms are preferable. For the MEGA telescope as well as for the prototype this is obviously the case. For example during the calibration of the MEGA prototype $15 \cdot 10^6$ events have been measured (Andritschke et al., 2004).

But list-mode has further advantages: Since no binned data-space is used, *all* information can be used with full precision as it has been measured. In addition to the data-space information, for MEGA this means using the first and the second interaction of the gamma-ray, the number of interactions, etc. Furthermore, it is easy to integrate different event types like untracked Compton events, tracked Compton events and pair events into one image. Finally, the detector geometry can easily be changed, which is especially useful for a prototype system under construction. Nevertheless list-mode has also one disadvantage: A pre-calculated response matrix does not exist. Instead it has to be calculated for each event individually during event reconstruction.

The selected imaging algorithm for the MEGA telescope is called List-Mode Maximum-Likelihood Expectation-Maximization (LM-ML-EM). Is has been originally developed for medical imaging (Wilderman et al., 1998). Its unbinned version is very similar to Richardson-Lucy-type algorithms.

3. THE LIST-MODE ALGORITHM

A Maximum-Likelihood Expectation-Maximization algorithm tries to determine an image by maximizing the expectation of the underlying likelihood function. This results in an image which fits the data best. The basic iterative reconstruction algorithm is given by:

$$\lambda_j^{(l+1)} = \frac{\lambda_j^{(l)}}{s_j} \sum_i^I \frac{v_i \cdot t_{ij}}{\sum_k t_{ik} \lambda_k} \qquad \forall j \in J \qquad (2)$$

with

I	data-space with all measured events
i	event index
J	image-space
j	image bin index
$\lambda_j^{(l)}$	image bin content a iteration level l
v_i	probability that the event i came from within the image-space (“visibility”)
t_{ij}	probability that a photon emitted from j is measure with the parameters of event i (“response”)
s_j	probability that an event emitted from j can be measured (“efficiency”)

An explanation how and why the algorithm works can be found in Lange et al. (1984) and Zoglauer (2000).

Compared to Wilderman et al. (1998), equation 2 contains the additional factor v_i, the visibility of event i. It stands for the probability that the event came from the selected image-space. This factor is in principle a reinterpretation of the original Y_i factor from the unbinned ML-EM algorithm (Wilderman et al., 1998), which represented the number of bins in the data-space element i: Especially for Compton events with large cone-sections in combination with small image-spaces, the event has a non-zero probability that it came from outside the image-space. As consequence only a fraction of the event is seen in the image, the factor v_i.

The original algorithm did not contain any criteria to stop the iterations. It simply converges asymptotically. The best images, however, are obtained shortly before convergence, when the width of the point sources corresponds to the intrinsic angular resolution of the detector. At this point the iteration is stopped.

4. RESPONSE OF THE MEGA TELESCOPE

Figures 2 show the four basic event types of the MEGA instrument:

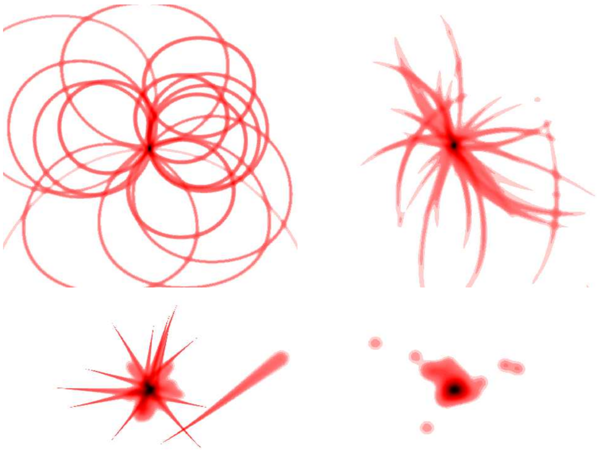

Figure 2. The 4 different events types of the MEGA telescope. Top Left: Untracked Compton events. Top Right: Tracked Compton events. Bottom Left: High energy tracked Compton events with incomplete absorption. Bottom right: Pair events.

In more than 50% of all interactions below 2 MeV the kinetic energy of the Compton recoil electron is not sufficient to produce a track. Therefore the origin can only be restricted to a cone (Fig. 2, top left). Its width is determined by energy and position measurement. Between 2 and 10 MeV most of the events have an electron track and most of them are completely absorbed (at least in the satellite geometry). Thus, the origin is restricted to an arc of the cone-section (Fig. 2, top right), whose length is determined by Molière scattering. Above 10 MeV most of the events are incompletely absorbed, but the direction of electron and scattered photon is well defined. Therefore, the origin of the photon can be restricted to the scattering plane and to the great circle between (reverse) electron and photon direction (Fig. 2, bottom left). Measured energies can further restrict this arc. The origin of pair events (Fig. 2, bottom right) is the bisecting line between electron and positron direction. The uncertainties are due to Molière scattering of electron and positron in the Silicon layers.

Three parameters describe the response in equation 2: the event response t_{ij}, the visibility v_i and the efficiency s_j. Figure 3 summarizes all probabilities which contribute to these parameters.

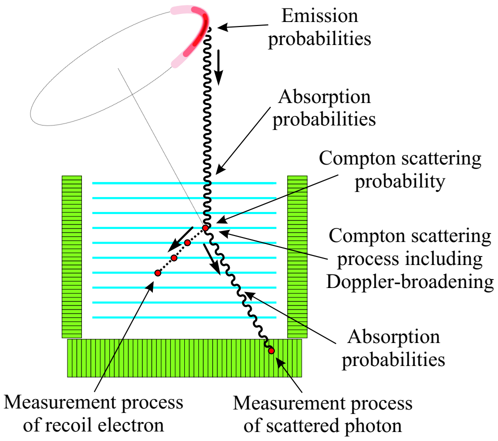

Figure 3. All probabilities which have to be considered during calculation of t_{ij} and s_j

The easiest way to determine the sensitivity s_j are simulations. This avoids the calculation of all the probabilities of figure 3. The visibility v_i is simply the length of the circle/arc in the image-space. Most efforts however needs the determination of the response t_{ij}: it is calculated by looping over all image-space bins and determines the probability that the measured photon was emitted there. The calculation has two steps: determine the absorption probability and the measurement process. In Wilderman et al. (1998) it has been shown that only the absorption probability of the initial photon passing through the detector to the first interaction position z_1 is important for the calculation for t_{ij}.

For Compton events, the measurement process is determined by two parameters: the profile p of the cone section and, in case of tracked events, the length l of the arc. The profile p of the cone-section is dominated by the energy measurement of electron E_e and scattered gamma-ray E_g, by Doppler-broadening, and by the spatial resolution of the detector: $p = p(E_g, E_e, z_1, z_2)$. For MEGA, the dominating factor is the energy resolution. Therefore it can be reduced to $p = p(E_g, E_e)$. Its shape can be approximated by combining a Lorentz-function with a Gaussian. The length l of the arc is mainly determined by direction and energy of the recoil electron. As consequence for electron energies below ~25 MeV it is dominated by Molière scattering and above by position resolution. So the length is given as $l = l(E_e, \vec{d_e})$. For the MEGA telescope its shape can be approximated by a Gaussian. Fig. 4 illustrates how both factors model the Compton response.

For low energies this response calculation gives the Compton circles/arcs of Fig. 2, top. With increasing energy, the length of the arc gets smaller, since the Molière scattering decreases, but the profile gets broader due to energy leakage. At high energies, when the energy measurement is rather incomplete,

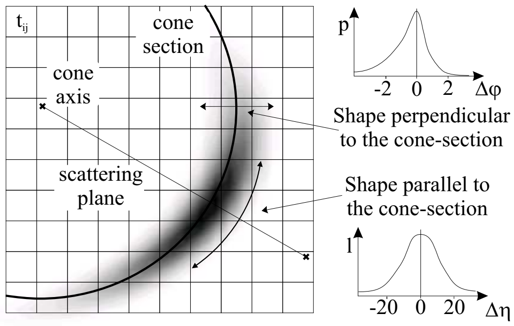

Figure 4. Modeling the measurement process: The cone-section is filled perpendicular to and along with two profiles: the first depends mainly on the energy measurement and the second on Molière scattering.

but the direction of the recoil electron is well known, the events have shapes like in Fig. 2, bottom left.

5. SELECTED RESULTS

Figure 5 shows a lab measurement with the MEGA prototype. A "radioactive" ring, produced by ^{88}Y on a rotating propeller with a radius of 7 cm and located 27 cm above the center of the tracker has been measured. The images contain ∼138,000 Compton events. An energy selection around the 0.9 MeV line of ^{88}Y has been applied (0.8-1.0 MeV). While the image at iteration 0 looks disk-like, with increasing number of iterations the ring structure gets visible (Fig. 5, right).

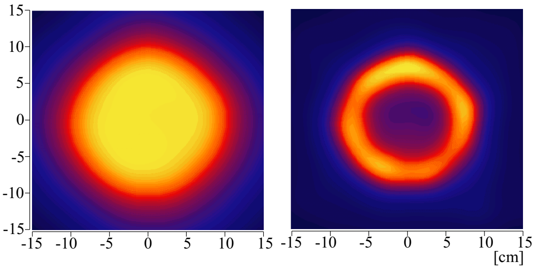

Figure 5. "Radioactive ring": In the image at iteration 0 (left), which is only a simple back projection of the events, the ring structure is hidden by cone circles and arcs. The ring structure gets visible after 50 iterations with the list-mode algorithm (right).

In a real space environment, a signal to background ratio of roughly 1:100 can be expected as seen by COMPTEL. A high background simulation with the MEGA satellite telescope is shown in Fig. 6. Events between 1 and 5 MeV (power-law spectrum) have been simulated with Geant4. 1.85 million uniformly distributed events intersect the reconstructed image. 286,000 events are within 3σ of the point spread function of the central point source, which contains 2,200 events. This results in a signal to background ratio of 1:130. No background estimation was applied.

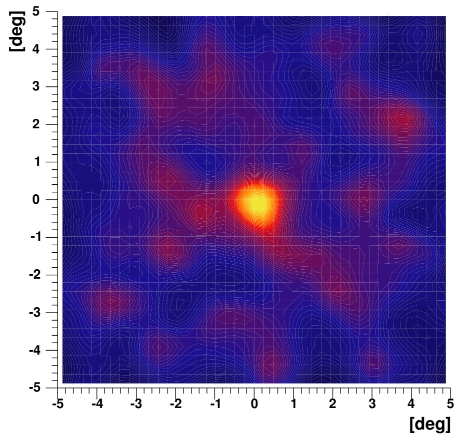

Figure 6. High background simulation with the MEGA telescope

More reconstructed images can be found in Zoglauer et al. (2004) (point sources ranging from 2 to 49 MeV) and Zoglauer et al. (2004a) (multiple point sources in the near field).

6. CONCLUSION

It has been shown that the List-Mode MLEM algorithm in combination with a good description of the response of the MEGA telescope allows to accurately retrieve positions of point sources as well as extended sources and retrieve point sources on high background. Furthermore the high flexibility of the implementation allows to easily exchange the detector geometry as well as to switch between near and far field. Thus, the algorithm is well suited for the use with the MEGA telescope.

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